

JYOTI PUBLIC SCHOOL, DHORKA

Holiday Homework
ASSIGNMENT, (2018-19)

CLASS – 12th

SUB-Mathematics.

- Q1. State true or false, using an example, the statement $|\vec{a}| = |\vec{b}| \Rightarrow \vec{a} = \vec{b}$.
- Q2. Given matrix $A = \begin{bmatrix} 3 & 2 & -1 \\ 4 & 1 & 6 \end{bmatrix}$. write its additive inverse.
- Q3. If $f: \mathbb{R} - \{0\} \rightarrow \mathbb{R} - \{0\}$ is given as $f(x) = \frac{3}{x}$, find $f^{-1}(x)$.
- Q4. Show that the relation S on \mathbb{R} , the set of real numbers, as $S = \{(x, y): x \leq y\}$ is reflexive but not symmetric.
- Q5. If matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, show that A is a zero of the polynomial $f(x) = x^2 - 4x + 7$.
- Q6. Using principle value, evaluate the following: $\cos^{-1}\left(\frac{2\pi}{3}\right) + \sin^{-1}\left(\frac{2\pi}{3}\right)$.
- Q7. Find the scalar and vector components of vector \vec{AB} with initial point $(2, 1)$ and terminal point $(-5, 7)$
- Q8. Find $\frac{dy}{dx}$, if $\tan^{-1}\left(\frac{a \cos x - b \sin x}{b \cos x + a \sin x}\right)$, $\frac{a}{b} \tan x > -1$.
- Q9. If function $f(x) = |x - 3| + |x - 4|$, show that f is not differentiable $x=3$ and $x=4$.
- Q10. Examine the continuity of the function $f(x) = \begin{cases} x - [x], & \text{if } x < 2 \\ 0, & \text{if } x = 2 \\ 3x - 5, & \text{if } x > 2 \end{cases}$ at $x=2$.
- Q11. Find the area of a rectangle $ABCD$ having vertices $A(\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k})$, $B(\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k})$, $C(-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k})$, $D(-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k})$.
- Q12. Find the interval (s) in which the function $f(x) = [x(x - 2)]^2$ is (i) strictly increasing (ii) strictly decreasing.
- Q13. Solve the matrix equation $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ 2 & 3 \end{bmatrix}$ using concept of inverse.
- Q14. Using properties of determinants, prove the following:
$$\begin{vmatrix} a + bx & c + dx & p + qx \\ ax + b & cx + d & px + q \\ u & v & w \end{vmatrix} = (1 - x^2) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$$
- Q15. Find the angle between the following pair of lines: $\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$ and $\vec{r} = (s - 1)\hat{i} + (2s - 1)\hat{j} + (2s - 1)\hat{k}$. Write the position vectors of two points through which the two given lines pass. Also, find a vector perpendicular to the direction vectors of the two given lines.
- Q16. Let $*$ be a binary operation on the set \mathbb{Q} of a rational number given by:
 $a * b = a + ab$. if $2 * x = 14$, find x .
- Q17. If matrix $A = \begin{bmatrix} 1 & -3 & 0 \end{bmatrix}$, then write AA' , where A' is transport of matrix A .
- Q18. If vectors $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$, are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to \vec{c} , then find the value of λ .
- Q19. If $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find x .
- Q20. Show that function f defined by $f(x) = \begin{cases} 3x - 2 & \text{if } 0 < x \leq 1 \\ 2x^2 - x & \text{if } 1 < x \leq 2 \\ 5x - 4 & \text{if } x > 2 \end{cases}$ is not differentiable at $x = 2$.
- Q21. Give set $A = \{1, 2, 3\}$ and a relation R in set A is defined as follows $R = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$. Write the ordered pair to be added to R to make it the smallest equivalence relation.

Q22. If $x = at^2$, $y = 2at$, find $\frac{d^2y}{dx^2}$ at $t = 2$.

Q23. If $A = \begin{bmatrix} 1 & 3 & -6 \\ 2 & 4 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 5 & 4 \\ 1 & 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 3 \\ -1 & 5 \\ 2 & 7 \end{bmatrix}$, find $2A - 3B + 5C$.

Q24. If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$ find x .

Q25. For any non-zero vectors \vec{a} and \vec{b} , prove that the vectors $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ and $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$ are perpendicular to each other.

Q26. Show that function $f(x) = |2x + 1|$ is not differentiable at $x = -\frac{1}{2}$.

Q27. Find A^{-1} , if $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and show $2A^{-1} = A^2 - 3I$.

Q28. If a, b, c are the roots of the equation $2x^3 - 4x^2 + 5x - 7 = 0$, find the value of the determinant

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}.$$

Q29. If $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 5\hat{j}$, $3\hat{i} + 2\hat{j} - 3\hat{k}$, and $\hat{i} + 6\hat{j} - \hat{k}$ are position vectors of points A, B, C, and D, then find the angle between \overrightarrow{AB} and \overrightarrow{CD} . Deduce that \overrightarrow{AB} and \overrightarrow{CD} are parallel or collinear.

Q30. Prove that $\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix}$ is a perfect square.